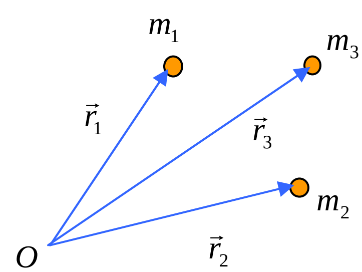
Center of Mass, Impulse, and Momentum

Center of Mass

The center of mass of a system of two particles is defined as the point with coordinate x_{CM} from the following formula

$$x_{CM} = \frac{x_1 m_1 + x_2 m_2}{m_1 + m_2}$$

In other words, center of mass is a point determined by the "mass weighted" average of their positions: It is a point between the particles, closer to the more massive one. An obvious conclusion is that if the masses are equal, then the center of mass is at the halfway point. For three and more particles, we may have to calculate not just the *x*-coordinate, but *y* and *z* as well. In general, the center of mass is defined by the following vector equation:



$$\vec{r}_{CM} = \frac{\Sigma m_i \vec{r}_i}{\Sigma m_i}$$

where m_i are all the individual masses. In component form:

$$x_{CM} = \frac{m_1 x_1 + m_2 x_2 + m_3 x_3 + ... + m_n x_n}{m_1 + m_2 + m_3 + ... + m_n}$$

$$y_{CM} = \frac{m_1 y_1 + m_2 y_2 + m_3 y_3 + ... + m_n y_n}{m_1 + m_2 + m_3 + ... + m_n}$$

$$z_{CM} = \frac{m_1 z_1 + m_2 z_2 + m_3 z_3 + ... + m_n z_n}{m_1 + m_2 + m_3 + ... + m_n}$$

A force of gravity on an extended object acts through its center of mass; hence, this point is sometimes called the center of gravity.

When a body or a system of particles is acted on by external forces, the center of mass moves as all the masses were concentrated at that point, and it is acted on by a resultant force which equals the sum of the external forces on the system.

Momentum

The following discussion refers to an isolated system of particles. An isolated system is one on which no external forces act. There are no isolated systems on Earth. However, if all external forces on the bodies in the system are balanced, the system can be considered isolated. For example, if the billiard balls are on a frictionless, flat surface, then their weight down is *always* balanced by an upward normal force.

The momentum of a body of mass *m* moving with velocity $\vec{v}$ is

$$\vec{p} \equiv m\vec{v}$$

Momentum is a vector quantity. The unit of momentum is kg·m/s.

In most problem-solving situations, a change in momentum results from the change in velocity. If the mass of a particle is constant, then

$$\Delta\vec{p} = \Delta(m\vec{v}) = m\Delta\vec{v}$$

From Newton's second law, $\sum \vec{F} = m\vec{a} = m\frac{\Delta\vec{v}}{\Delta t} = \frac{\Delta\vec{p}}{\Delta t}$ (or $\frac{d\vec{p}}{dt}$)

The rate of change of momentum of an object is equal to the net force acting on it. This is a restatement of Newton's second law in terms of momentum. This form of the second law corresponds to Newton's original formulation, and it also holds for changing mass.

Impulse of a force is defined as $\vec{J} = \vec{F}_{avg}\Delta t$ where $\vec{F}_{avg}$ is the average force acting, then from $\Sigma\vec{F} = \frac{\Delta\vec{p}}{\Delta t}$ making $\Sigma\vec{F}\Delta t = \Delta\,\vec{p}$ we get $\vec{J} = \Delta\vec{p}$. For example, the impulse of a force is equal to the change in momentum it causes.

If $\Sigma\vec{F} = 0, \quad \Delta\vec{p} = 0$. If the net force on the particle is zero, its momentum cannot change; hence, its velocity cannot change. This is just a restatement of Newton's first law.